by Lawrence S. Speidell, Deborah H. Miller and James R. Ullman

Portfolio Optimization: A Primer

Portfolio optimization is a procedure for measuring and controlling portfolio risk and expected return. At its simplest, portfolio optimization is basically diversification—reducing portfolio risk by combining assets whose specific risks offset each other. But optimization usually also takes into consideration the correlations between assets—the extent to which their prices tend to move together. By combining stocks in different groups whose price moves tend to complement one another, an optimizer can build a portfolio that offers the highest level of return for each level of risk.

Of course, the "optimal" portfolio for a given client will depend upon the client's perception of risk. Risk should thus be measured relative to the index the client uses as a performance benchmark. This may be the S&P 500, the Value Line Index or a "normal portfolio" constructed to typify a particular investment style. The degree to which the actual portfolio differs from the benchmark determines the portfolio's risk. The riskier the portfolio, the higher the return it should achieve over the long term.

PORTFOLIO OPTIMIZATION is the measurement and control of portfolio risk and expected return. It is an important part of the investment process, but the associated jargon can obscure rather than illuminate the investment manager's role in it. This article tries to shed some light on the subject.

Risk and Return

In its simplest terms, selecting an investment portfolio is a bit like selecting a beverage. As Figure A shows, each beverage starts with water and moves away from it in a different direction. Sometimes we choose how potent the brew should be, and other times we simply take it as it comes. With tea, for example, we can decide precisely how long to let it steep.

The diagram for investment management (Figure B) is similar. The neutral territory in the middle represents the "market," and each of the major investment styles radiates out in a different direction. Just as tea may be weak or strong, any investment style may be followed to a greater or lesser extent.

"Core" portfolios closest to the market, for example, are basically index funds designed to track market movements. Core portfolios further from the market devote increasingly larger percentages of their assets to the pursuit of increased return through income, growth, value, contrary or other strategies. Most institutional investors diversify across styles by hiring several managers and then diversify within styles by hiring specialty managers for small, high-risk/high-reward portfolios while core managers handle the rest.

The risk of an equity investor is measured by deviation from the market return. The risk-reward tradeoff is a function of the distance from the middle, rather than the choice of any particular investment style. A portfolio can be aggressive or conservative growth, aggressive or conservative value, and so on. Portfolio optimization does not influence the direction in which a portfolio moves. Instead, it determines the distance: How far are we going to move away from the market? How much risk are we willing to take? How much reward are we going to aim for? Optimization makes sure that, at each level of reward, the portfolio has the lowest possible risk.

Diversification

At its most basic level, optimization is diversification.[1] If we own one business that rents golf

Lawrence Speidell, Deborah Miller and James Ullman are Trustees of Batterymarch Financial Management.

1. Footnotes appear at end of article.

Figure A Beverage Choices

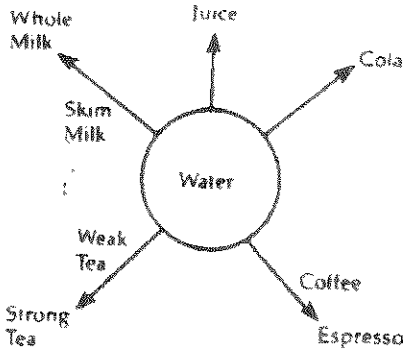
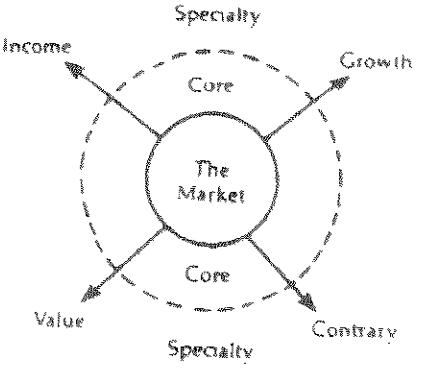


Figure B Investment Choices



carts and another that rents umbrellas, we won't have to worry about the weather, because one business will always be doing relatively well. In building a diversified portfolio, investors are dealing with two important characteristics—specific risk and correlation.

Specific risk is the exposure of a security to events that affect it alone, as opposed to market risk, which is shared by all securities in the same market or asset class. (See the appendix for a technical explanation.) Specific risk caused the decline in Union Carbide in 1984 (due to the Bhopal disaster); market risk caused all stocks to decline on October 19, 1987.

Correlation is the extent to which stock prices move together. If energy prices drop, for example, oil stocks may all decline, while airline stocks may rise, as investors expect oil profits to drop while airline profits increase. The oil stocks may be said to be positively correlated with each other (their prices move together) and positively correlated with energy prices (they move in the same direction). Airline stocks are positively correlated with each other, but negatively correlated with energy prices (they move in opposite directions).

Statistically speaking, correlations run from +1 (when two securities have always moved in the same direction) to −1 (when they have moved in exactly opposite directions). A zero correlation means that their movements have been completely independent of each other. Figure C illustrates portfolios of two stocks with different correlations; the length of the arrows represents risk and their direction represents correlation.

The first case shows our golf cart and umbrella businesses. Here the net total portfolio risk, P, is zero because the correlation of the two securities A and B is −1. In the second case, the two securities are completely uncorrelated; their correlation is zero, and the vectors for the two form a 90-degree angle. The risk of an equal-weighted portfolio of the two securities is less than the risk of either security; this is shown by the shorter length of arrow P. (These two cases are detailed in the appendix.) Finally, the third and fourth cases show the portfolio risk that results from combining securities with other correlation characteristics.

Diversification will always reduce the specific risk of a portfolio, but it may not make optimal use of correlation. That is where optimization comes in.

Figure C Correlation Wheels

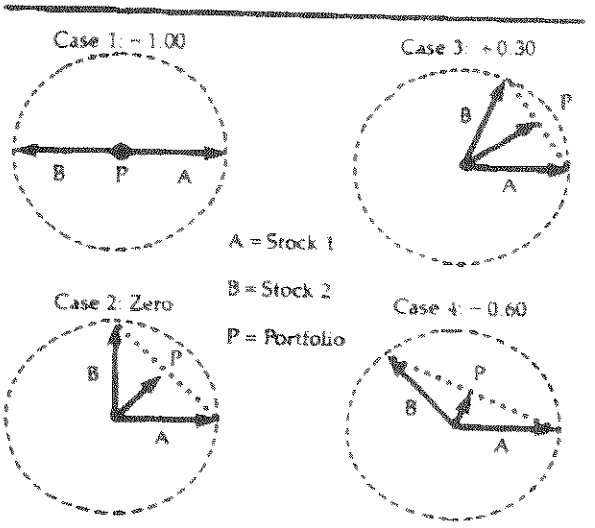
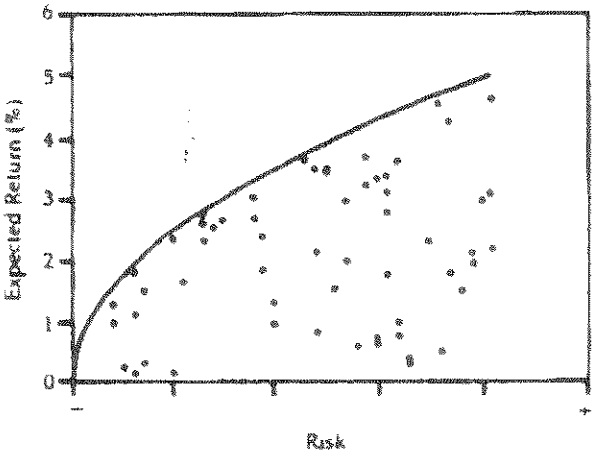


Figure D Efficient Frontier



The Efficient Frontier

The most important steps in investment management are the analysis and ranking of securities. The manager's goal is to identify those stocks that have the highest expected returns. If this task is not done properly, then the portfolio will be disappointing no matter how carefully it is optimized. Of course, if it is done perfectly, then optimization won't matter much either, because perfect foreknowledge of returns would allow us to buy the one best stock and forget about the rest.

In the real world, in the absence of perfect foresight, we need to weigh the expected return of each investment against its risk. Figure D shows a traditional diagram of securities plotted by their expected returns and risks. The cloud of stocks trends up to the right because risk and reward (expected return) are positively correlated in a rational market; that is, higher-risk stocks offer higher returns.

There are two important observations to make about this chart. One is that there are many stocks at each level of risk, but only one has the highest return. (It is all too easy to take high risks without getting commensurate rewards, especially in the short run.) The second point is that the law of diminishing returns is evident: Beyond a certain point, as risk increases, the associated increases in return become smaller.

The efficient frontier curve drawn in the figure represents the highest returns available at any given level of risk. It lies slightly above and to the left of the highest securities because of the benefits of correlation. When we combine two stocks with a correlation of less than 1, the reward is the average of their expected returns, but the risk is less than the average of their risks. By combining stocks in different groups, whose price moves tend to complement one another, an optimizer can build the most efficient portfolio, the one that offers the highest reward for each level of risk.

Optimizers and Allocators

In practice, most institutional money managers make their optimization tradeoffs on the basis of some simple measures of risk, usually market risk (beta) and industry diversification. More quantitative managers introduce additional risk measures based on common characteristics such as price/book value, size, financial leverage and earnings stability.

The simplest computer optimizers are really allocators; they ensure that a portfolio is well diversified and has the desired representation in each industry, each size range, each beta range and so on. The equations used in these allocations are simply linear "maximizers." More sophisticated quadratic optimizers take advantage of the correlation characteristics discussed above. They may use a limited covariance matrix of industry and risk characteristics or more complex matrixes that take into account any number of individual stock characteristics. The appendix provides an example of the latter type of optimizer.

Optimizing Relative to What?

Risk should be measured relative to the particular index each client uses as a performance benchmark. The S&P 500 is the most widely used measure, but larger composites, such as the Value Line index, or a "normal portfolio" constructed to typify a particular investment style may be used.

To measure the extent to which a portfolio differs from the chosen benchmark, we use a statistical measure called R-squared, which can range from zero to 1.00 and indicates how much of the movement in the portfolio is explained by movement in the benchmark. (See the appendix.) A perfect index fund would have an R-squared of 1.00, indicating that 100 per cent of its movement is explained by movement in the index it is trying to track. An R-squared of zero would mean that there is no relationship be-